

Interaction of Electromagnetic S-Wave with a Metal Film Located Between Two Dielectric Mediums

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Generalization of the theory of interaction of electromagnetic S – wave with a metal film on a case of the film concluded between two dielectric environments is developed.

Key words: degenerate plasma, dielectric permeability, metal films, dielectric media, transmittance, reflectance, absorptance.

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Introduction

Problem of interaction of an electromagnetic wave with a metal film long time draws to itself attention [1] – [9]. It is connected with theoretical interest to this problem, and with numerous practical applications.

Nowadays there is the theory of interaction of an electromagnetic wave with plasma layer in the case when electrons reflection from a film surface has a specular character [1] – [5]. In these works it was considered the freely hanging films in air. In other words it was considered the case, when dielectric permeability of the environments surrounding a film is equal to unit.

However in overwhelming majority of cases it is not so [6]. As a rule in practice one deals with the films located on some dielectric substrate. May be also cases when the metal film is located between two dielectric environments. Generalisation of the available theory of interaction of electromagnetic radiation with metal film on a such situations will be the purpose of our work.

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Problem formulation

Let us consider the thin layer of metal located between two dielectric environments. We will assume, that these environments are not magnetic. Their dielectric permeability we will designate through ε_1 and ε_2 . Let us designate these environments by the first and the second media according to. We consider the case, when the first media are not absorbing. Let's on a film from the first media the electromagnetic wave falls. Incidence angle we will designate θ . Let us assume, that a vector of electric field of the electromagnetic waves is parallel to a layer surface. Such a wave is called H – wave [3] (or S – wave [1]).

We take the Cartesian system of coordinates with the beginning of coordinates on the surface of a layer adjoining on the first media. We will direct axis x into a layer. And we will direct axis y parallel to electric field vector of electromagnetic wave.

Components of electric and magnetic field vectors we will search in the form

$$E_y(x, z, t) = E_y(x)e^{-i\omega t + ik_z z},$$

and

$$H_x(x, z, t) = H_x(x)e^{-i\omega t + ik_z z}, \quad H_z(x, z, t) = H_z(x)e^{-i\omega t + ik_z z}.$$

We denote the thickness of the layer by d .

Let us designate by $Z^{(1)}$ an impedance on the bottom layer surfaces at antisymmetric on electric field configurations of external fields. It is the case 1, when

$$E_y(0) = -E_y(d), \quad H_z(0) = H_z(d), \quad \frac{dE_y(0)}{dx} = \frac{dE_y(d)}{dx}. \quad (1)$$

Let us designate by $Z^{(2)}$ an impedance on the bottom layer surfaces at symmetric on electric field configurations of external fields. It is the case 2, when

$$E_y(0) = E_y(d), \quad H_z(0) = -H_z(d), \quad \frac{dE_y(0)}{dx} = -\frac{dE_y(d)}{dx}. \quad (2)$$

Out of a layer it is possible to present electric fields in the following form

$$E_y^{(j)}(x) = \begin{cases} a_j h_j e^{ik_{2x}(x-d)} + b_j h_j e^{-ik_{2x}(x-d)}, & x > d, \\ h_j e^{ik_{1x}x} + p_j h_j e^{-ik_{1x}x}, & x < 0, \end{cases} \quad j = 1, 2. \quad (3)$$

Indexes "1" and "2" at factors a, b, h, p and field projections $E_y(x)$ correspond to the first and to the second cases accordingly.

Thus the impedance in both cases is defined as follows (see, for example, [12] and [11])

$$Z^{(j)} = \frac{E_y^{(j)}(-0)}{H_z^{(j)}(-0)}, \quad j = 1, 2.$$

Transmittance, reflectance and absorptance

From Maxwell equations follows that [11]

$$Z^{(j)} = \frac{i\omega}{c} \frac{E_y^{(j)}(-0)}{\frac{dE_y^{(j)}(-0)}{dx}}, \quad j = 1, 2.$$

Here c is the speed of light.

The symmetry of a field for the first case with use (1) and (3) leads to the following relations

$$-a_1 - b_1 = 1 + p_1, \quad k_{2x}a_1 - k_{2x}b_1 = k_{1x} - k_{1x}p_1. \quad (4)$$

Solving system (4), we have

$$a_1 = \frac{k_{1x}}{2k_{2x}}(1 - p_1) - \frac{1 + p_1}{2}, \quad b_1 = -\frac{k_{1x}}{2k_{2x}}(1 - p_1) - \frac{1 + p_1}{2}. \quad (5)$$

The symmetry of a field for the second case with use (2) and (3) leads to the following relations

$$a_2 + b_2 = 1 + p_2, \quad k_{2x}a_2 - k_{2x}b_2 = -k_{1x} + k_{1x}p_2. \quad (6)$$

The solution of the system (6) has the following form

$$a_2 = -\frac{k_{1x}}{2k_{2x}}(1 - p_2) + \frac{1 + p_2}{2}, \quad b_2 = \frac{k_{1x}}{2k_{2x}}(1 - p_2) + \frac{1 + p_2}{2}. \quad (7)$$

Let us consider the following configuration of the field

$$E_y(x) = b_2 h_2 E_y^{(1)}(x) - b_1 h_1 E_y^{(2)}(x).$$

The field $E_y(x)$ has the following structure

$$E_y(x) = \begin{cases} (a_1 b_2 - a_2 b_1) h_1 h_2 e^{ik_{2x}(x-d)}, & x > d, \\ (b_2 - b_1) h_1 h_2 e^{ik_{1x}x} + (p_1 b_2 - p_2 b_1) h_1 h_2 e^{-ik_{1x}x}, & x < 0. \end{cases}$$

Thus, electric field corresponds to the electromagnetic wave falling on film from negative semispace. A wave partially passes through a film, and it is partially reflected.

Thus taking into account the equations (5) and (7) we have

$$\begin{aligned} a_1 b_2 - a_2 b_1 &= \frac{k_{1x}}{k_{2x}}(p_2 - p_1), \\ b_2 - b_1 &= \frac{k_{1x}}{k_{2x}}(1 - \bar{p}) + 1 + \bar{p}, \\ p_1 b_2 - p_2 b_1 &= \bar{p} + p_1 p_2 + \frac{k_{1x}}{k_{2x}}(\bar{p} - p_1 p_2), \end{aligned}$$

where

$$\bar{p} = \frac{p_1 + p_2}{2}.$$

The quantities p_1, p_2 may be presented as

$$p_j = \frac{ck_{1x}Z^{(j)} - \omega}{ck_{1x}Z^{(j)} + \omega}, \quad j = 1, 2.$$

The reflection coefficient of an electromagnetic wave is equal to

$$R = \left| \frac{\bar{p}(k_{1x} + k_{2x}) + 2(k_{2x} - k_{1x})p_1p_2}{k_{1x} + k_{2x} + (k_{2x} - k_{1x})\bar{p}} \right|^2.$$

We may express the quantity k_{2x} through k_{1x} and dielectric permeabilities ε_1 and ε_2 [11]

$$k_{2x} = \frac{\omega}{c} \sqrt{\varepsilon_2 - \varepsilon_1 \sin^2 \theta}, \quad k_{1x} = \frac{\omega}{c} \sqrt{\varepsilon_1} \cos \theta.$$

We can present the relations for p_1 and p_2 considering last expression in the following form

$$p_j = \frac{\sqrt{\varepsilon_1} \cos \theta Z^{(j)} - 1}{\sqrt{\varepsilon_1} \cos \theta Z^{(j)} + 1}, \quad j = 1, 2.$$

Average value of energy flux of the electromagnetic wave $\langle \mathbf{S} \rangle$ is equal to [7]

$$\langle \mathbf{S} \rangle = \frac{c}{16\pi} \{ [\mathbf{E}\mathbf{H}^*] + [\mathbf{E}^*\mathbf{H}] \}.$$

Here the asterisk designates complex conjugation.

According to Maxwell equations the magnetic field of the wave \mathbf{H} may be presented in the form [7]

$$\mathbf{H} = \frac{c}{\omega} [\mathbf{k}\mathbf{E}].$$

According to these relations for quantity $\langle S_x \rangle$ we obtain the result

$$\langle S_x \rangle = \frac{c}{16\pi} (E_y H_z^* + E_y^* H_z) = \frac{c^2}{8\pi\omega} |E_y|^2 \operatorname{Re}(k_x).$$

The reflection coefficient R thus can be written down as

$$R = \left| \frac{\sqrt{\varepsilon_2 - \varepsilon_1 \sin^2 \theta}(\bar{p} + p_1p_2) + \sqrt{\varepsilon_1} \cos \theta(\bar{p} - p_1p_2)}{\sqrt{\varepsilon_2 - \varepsilon_1 \sin^2 \theta}(1 + \bar{p}) + \sqrt{\varepsilon_1} \cos \theta(1 - \bar{p})} \right|^2.$$

Let us introduce the quantity $\varepsilon_{12} = \frac{\varepsilon_2}{\varepsilon_1}$, characterising the ratio of coefficients ε_2 and ε_1 . Then the relation for reflection coefficient will be rewritten in the form

$$R = \left| \frac{\sqrt{\varepsilon_{12} - \sin^2 \theta}(\bar{p} + p_1p_2) + \cos \theta(\bar{p} - p_1p_2)}{\sqrt{\varepsilon_{12} - \sin^2 \theta}(1 + \bar{p}) + \cos \theta(1 - \bar{p})} \right|^2. \quad (8)$$

We can to present the coefficient of transmission T in the form

$$T = \frac{\operatorname{Re}(k_{2x})}{k_{1x}} \left| \frac{a_1 b_2 - a_2 b_1}{b_2 - b_1} \right|^2.$$

The last expression may be rewritten in the form

$$T = k_{1x} \operatorname{Re}(k_{2x}) \left| \frac{p_2 - p_1}{k_{1x}(1 - \bar{p}) + k_{2x}(1 + \bar{p})} \right|^2,$$

or in the explicit form

$$T = \cos \theta \operatorname{Re}(\sqrt{\varepsilon_{12} - \sin^2 \theta}) \left| \frac{p_2 - p_1}{\sqrt{\varepsilon_{12} - \sin^2 \theta}(1 + \bar{p}) + \cos \theta(1 - \bar{p})} \right|^2. \quad (9)$$

For transparent environments the quantities ε_1 and ε_2 are real. From the obtained formula for transmission coefficient it is clear, that in this case, when

$$\sin^2 \theta \geq \frac{\varepsilon_2}{\varepsilon_1}$$

the transmission coefficient is equal to zero, as thus

$$\operatorname{Re}(\sqrt{\varepsilon_{12} - \sin^2 \theta}) = 0.$$

It corresponds to full internal reflection.

We note that by $\sin^2 \theta \rightarrow \varepsilon_2/\varepsilon_1$ the transmission coefficient $T \rightarrow 0$. The reflection coefficient R by $\theta \rightarrow \pi/2$ tends to 1.

Now we can find the absorption coefficient A according to the formula

$$A = 1 - T - R. \quad (10)$$

Coefficients of transmission T and reflection R of the electromagnetic waves by layer at $\varepsilon_1 \rightarrow 1, \varepsilon_2 \rightarrow 1$ transforms in earlier known expressions [1], [8]

$$T = \frac{1}{4} |p_1 - p_2|^2, \quad R = \frac{1}{4} |p_1 + p_2|^2.$$

Let us consider the case of specular reflection of electrons from a film surface. Then for the quantities $Z^{(j)}$ ($j = 1, 2$) the following relations are valid [1]

$$Z^{(j)} = \frac{2i\Omega}{W} \sum_{n=-\infty}^{n=\infty} \frac{1}{\Omega^2 \varepsilon_{tr} - Q^2}, \quad j = 1, 2,$$

where

$$W = W(d) = \frac{\omega_p d}{c} \cdot 10^{-7}.$$

And the thickness of the film d is measured in nanometers, for $Z^{(1)}$ summation is conducted on odd n , and for $Z^{(2)}$ on the even.

Here

$$\varepsilon_{tr} = \varepsilon_{tr}(q_1, \Omega), \quad \Omega = \frac{\omega}{\omega_p}, \quad \mathbf{q}_1 = \frac{v_F}{c} \mathbf{Q},$$

$$\mathbf{Q} = (Q_x, 0, Q_z), \quad Q_x = \frac{\pi n}{W(d)}, \quad Q_z = \sqrt{\varepsilon_1} \Omega \sin \theta,$$

\mathbf{q}_1 is the dimensionless wave vector, $\mathbf{q} = \frac{\omega_p}{v_F} \mathbf{q}_1$ is the dimensional wave vector, $\varepsilon = \frac{\nu}{\omega_p}$ and ν are dimensionless and dimensional frequencies of electron collisions accordingly, module of vector \mathbf{q}_1 is equal to

$$q_1 = \frac{v_F}{c} \sqrt{\frac{\pi^2 n^2}{W^2(d)} + \varepsilon_1 \Omega^2 \sin^2 \theta}.$$

Here ω_p is the plasma (Langmuir) frequency, ε_{tr} is the transverse dielectric permeability, which may be presented in explicit form by formula

$$\varepsilon_{tr} = 1 - \frac{3}{4\Omega q_1^3} \left[2(\Omega + i\varepsilon)q_1 + [(\Omega + i\varepsilon)^2 - q_1^2] \ln \frac{\Omega + i\varepsilon - q_1}{\Omega + i\varepsilon + q_1} \right].$$

We transform now the functions $Z^{(1)}$ and $Z^{(2)}$

$$Z^{(1)} = \frac{4i\Omega}{W(d)} \sum_{n=1}^{+\infty} \frac{1}{\Omega^2 \varepsilon_{tr}(\Omega, \varepsilon, d, 2n-1, \theta, \varepsilon_1) - Q(\Omega, d, 2n-1, \theta, \varepsilon_1)},$$

$$Z^{(2)} = \frac{2i\Omega}{W(d) [\Omega^2 \varepsilon_{tr}(\Omega, \varepsilon, d, 0, \theta, \varepsilon_1) - Q(\Omega, d, 0, \theta, \varepsilon_1)]} +$$

$$+ \frac{4i\Omega}{W(d)} \sum_{n=1}^{+\infty} \frac{1}{\Omega^2 \varepsilon_{tr}(\Omega, \varepsilon, d, 2n, \theta, \varepsilon_1) - Q(\Omega, d, 2n, \theta, \varepsilon_1)}.$$

Analys of results

We consider a fim of sodium. Then [1] $\omega_p = 6.5 \times 10^{15} \text{ sec}^{-1}$, $v_F = 8.52 \times 10^7 \text{ cm/sec}$.

Let us carry out graphic research of coefficients of transmission, reflection and absorption, using formulas (9), (8) and (10).

Let us fix dimensionless frequency of electron collisions for all figures $\varepsilon = 0.001$. It means, that dimensional frequency of electron collisions is equal: $\nu = 0.001\omega_p$.

On Figs. 1 – 6 the normal falling of an electromagnetic wave on a film from the first the dielectric environment is considered, on Fig. 7 dependence of reflectance on a thickness of a film is considered, on Figs. 8 – 10 dependence of coefficients T, R, A on an angle of incidence of an electromagnetic wave is considered.

On Figs. 1 – 3 the system air - film - glass is considered ($\varepsilon_1 = 1, \varepsilon_2 = 4$).

On Fig. 1 dependence of transmittance coefficient on frequency of an electromagnetic wave $T = T(\Omega, d)$ is represented. We consider this coefficient at the various values of a thickness of the film: $d = 100$ nm (curve 1), $d = 150$ nm (curve 2) and $d = 200$ nm (curve 3).

Plots on Fig. 1 show the monotonous increasing of transmittance coefficient at frequencies less than plasma frequency: $\omega < \omega_p$, i.e. at $\Omega < 1$. In region of the resonant frequencies (i.e. at $\Omega > 1$) transmittance coefficient has oscillatory (nonmonotonic) character. Than the thickness of the film is more, the oscillatory character is more strongly expressed, i.e. local maxima and minima alternate more often.

On Fig. 2 dependence of reflectance coefficient on dimensionless frequency of electromagnetic wave at the same values of film thickness is represented.

Curves 1,2,3 correspond to the same values thickness of a film, as on Fig. 2. Reflectance coefficient decreases monotonously at $\Omega < 1$ at all values of a thickness of the film. At $\Omega > 1$ we have some local extrema. Their locations depend on the film thickness.

On Fig. 3 dependence of absorption coefficient on dimensionless frequencies Ω at the same values of a film thickness is represented. The absorptance has a local maximum in neighborhood of plasma frequency, i.e. at $\Omega = 1$. The value of a local maximum depends on the film thickness.

On Figs. 4 – 6 dependence of coefficients T, R, A on dimensionless frequency of an electromagnetic wave is represented. On these Figs. the film thickness is equal to $d = 100$ nm. As the first dielectric environment the air is considered. As the second dielectric environment (substrate) consecutive we take glass (curve 1), mica (curve 2) and ceramics radio engineering (curve 3).

Plots on Fig. 4 show, that with growth quantity of dielectric permeability the value of transmittance decrease irrespective of dimensionless oscillation frequency of electromagnetic wave. Besides, this coefficient monotonously increases before the local maximum (laying in region $1 < \Omega < 2$) irrespective of quantity of dielectric permeability.

Plots on Fig. 5 show, that with growth of the dielectric permeability the values of reflectance increase irrespective of dimensionless frequency of electromagnetic wave. The reflectance has the local minimum in the same region of $1 < \Omega < 2$. Besides, this coefficient monotonously decreases before the local minimum irrespective of quantity of dielectric permeability.

Plots on Fig. 6 show, that near to the point $\Omega = 1$ values of absorptance are close to each other independent on quantity of dielectric permeability. Change of this coefficient has nonmonotonic character. In region of $0 < \omega < 1$ the absorptance has a minimum, and in region of $1 < \Omega < 2$ this coefficient has a maximum. At frequencies less than $\Omega = 1$

quantities of coefficient A increase with decreasing of the values of dielectric permeability, and at $\Omega > 1$ this situation varies on the opposite.

On Fig. 7 the dependence of reflectance as function of thickness of a film in region of $100 \text{ nm} < d < 200 \text{ nm}$ is represented. As the first the dielectric environment air is considered. As the second the dielectric environment (substrate) sequentially we take glass (curve 1), mica (curve 2) and ceramics radio engineering (curve 3). Plots show, that with growth of quantity of dielectric permeability the quantity of reflectance grows independently of the film thickness.

In last Figs. 8 – 10 the dependence of coefficients T, R, A as functions on angle of incidence of the electromagnetic waves is represented. Curves 1,2,3 correspond to values of a thickness of a film $d = 100 \text{ nm}, 150 \text{ nm}, 200 \text{ nm}$. Plots in these drawings show, that values of transmittance decrease irrespective of thickness of a film (Fig. 8). Plots on Fig. 9 show growth of reflectance with growth of the film thickness, and a drawing on Fig. 10 show monotonous decrease of absorptance, and with growth of a film thickness the values of absorptance increase also.

Conclusion

In the present work generalisation of the theory of interaction of electromagnetic radiation (S - waves) with a metal film on a case the film concluded between two various dielectric environments is represented.

Formulas for transmittance, reflectance and absorptance of an electromagnetic wave are obtained.

The graphic representation of these coefficients as a functions of frequency of an electromagnetic wave, angle of incidence of the electromagnetic waves on the film, thickness of the film and character (quantity) of the second dielectric environment (substrate) is fulfilled.

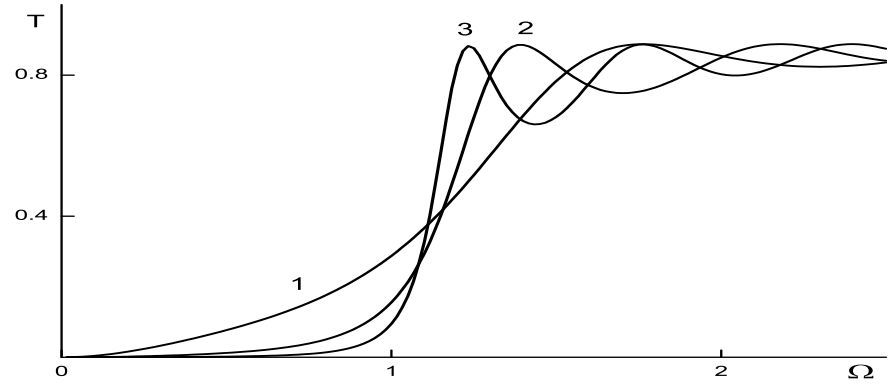


Figure 1. Transmittance, air-film-glass, curves 1,2,3 correspond to values $d = 100, 150, 200$ nm, $0 \leq \Omega \leq 2.5$, $\nu = 0.001\omega_p$, $\theta = 0^\circ$.

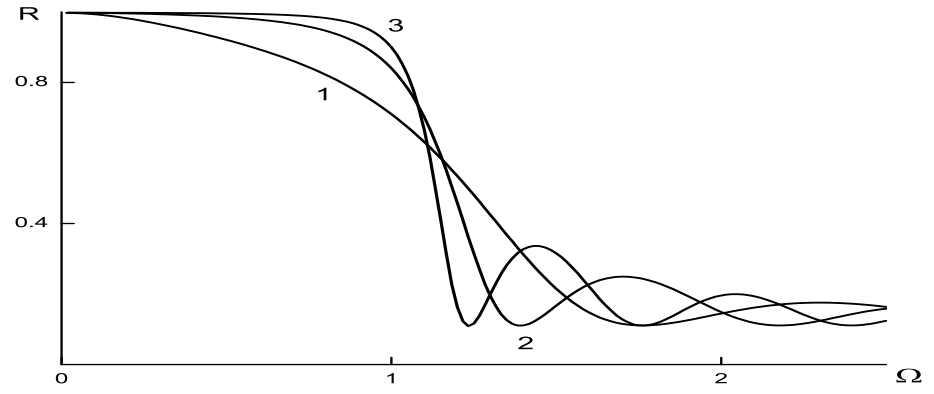


Figure 2. Reflectance, air-film-glass, curves 1,2,3 correspond to values $d = 100, 150, 200$ nm, $0 \leq \Omega \leq 2.5$, $\nu = 0.001\omega_p$, $\theta = 0^\circ$.

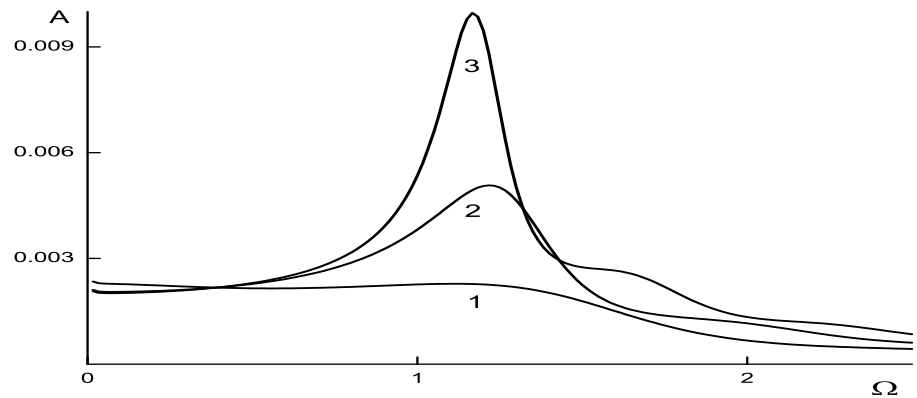


Figure 3. Absorptance, air-film-glass, curves 1,2,3 correspond to values $d = 100, 150, 200$ nm, $0 \leq \Omega \leq 2.5$, $\nu = 0.001\omega_p$, $\theta = 0^\circ$.

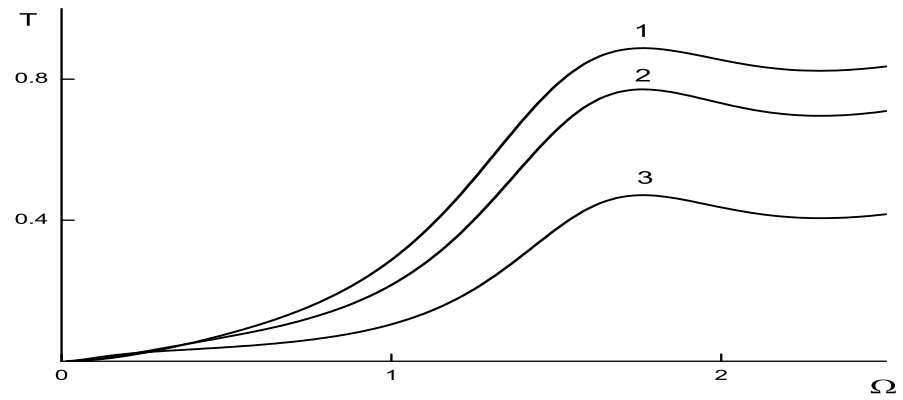


Figure 4. Transmittance, $d = 100$ nm, $\varepsilon_1 = 1$ – air, curves 1,2,3 correspond to values $\varepsilon_2 = 4, 8, 40$ (glass, mica, ceramic radiotechnical), $0 \leq \Omega \leq 2.5$, $\nu = 0.001\omega_p$, $\theta = 0^\circ$.

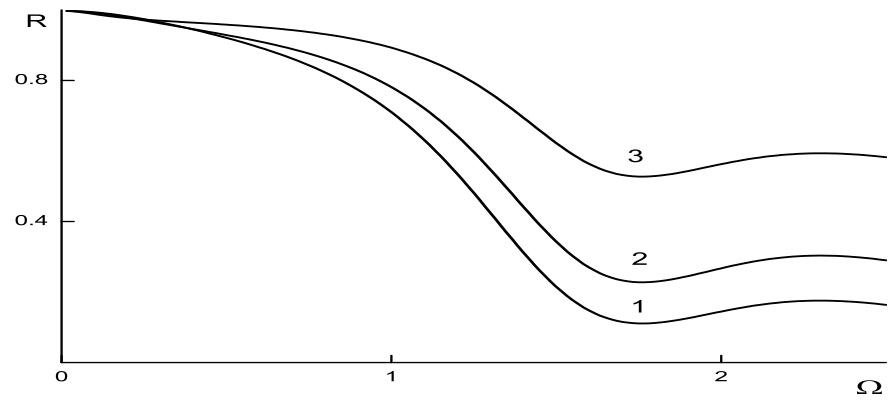


Figure 5. Reflectance, $d = 100$ nm, $\varepsilon_1 = 1$ – air, curves 1,2,3 correspond to values $\varepsilon_2 = 4, 8, 40$ (glass, mica, ceramic radiotechnical), $0 \leq \Omega \leq 2.5$, $\nu = 0.001\omega_p$, $\theta = 0^\circ$.

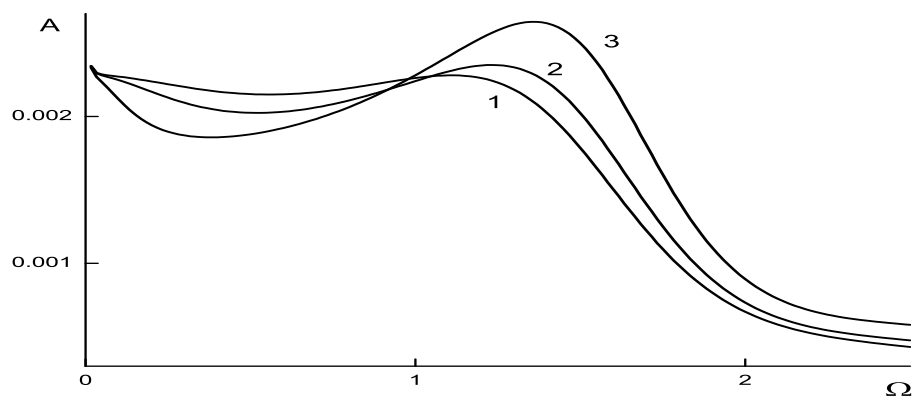


Figure 6. Absorptance, $d = 100$ nm, $\varepsilon_1 = 1$ – air, curves 1,2,3 correspond to values $\varepsilon_2 = 4, 8, 40$ (glass, mica, ceramic radiotechnical), $0 \leq \Omega \leq 2.5$, $\nu = 0.001\omega_p$, $\theta = 0^\circ$.

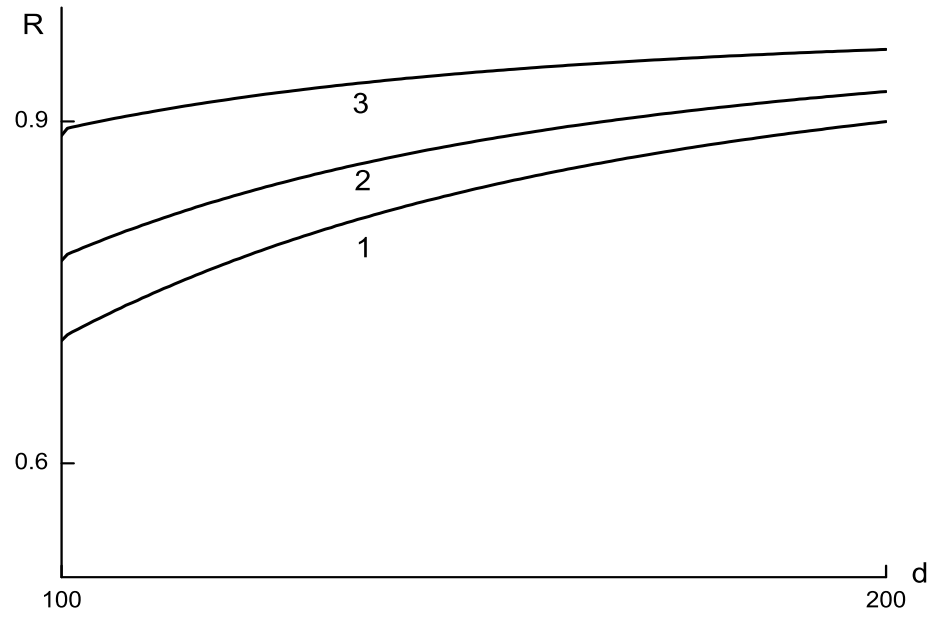


Figure 7. Reflectance , $\Omega = 1$, $\varepsilon_1 = 1$ – air, curves 1,2,3 correspond to values $\varepsilon_2 = 4, 8, 40$ (glass, mica, ceramic radiotechnical), $100 \leq d \leq 200$, $\nu = 0.001\omega_p$, $\theta = 0^\circ$.

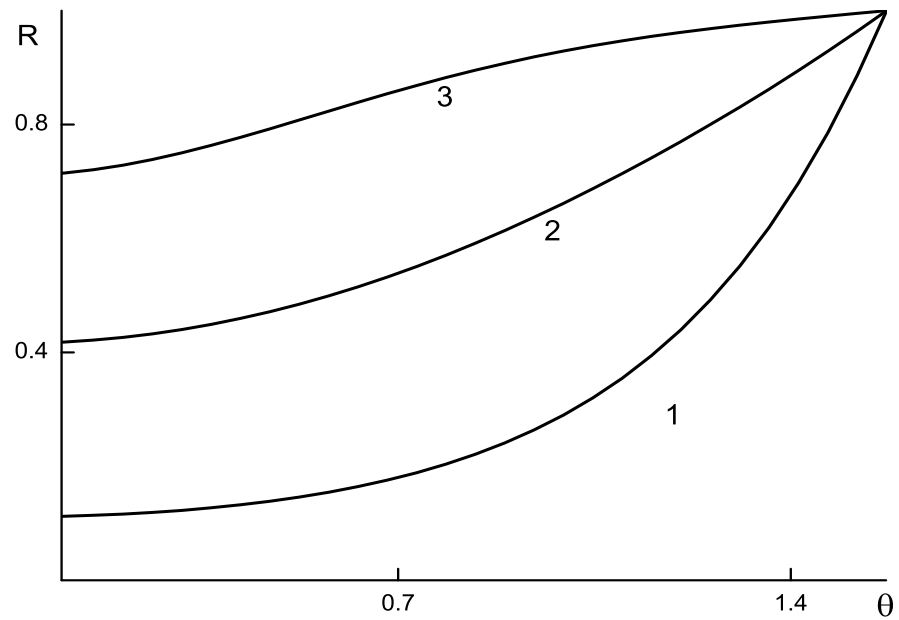


Figure 8. Transmittance, $\Omega = 1$, $\varepsilon_1 = 1$ – air, $\varepsilon_2 = 4$ – glass, curves 1,2,3 correspond to values $d = 10, 50, 100$, $\nu = 0.001\omega_p$, $0 \leq \theta \leq \pi/2$.

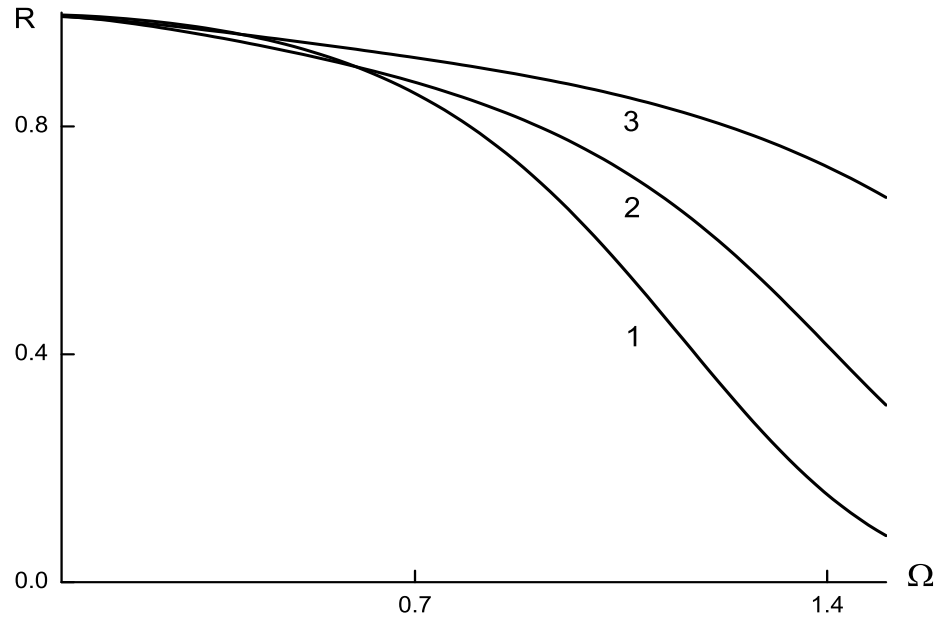


Figure 9. Reflectance , $\Omega = 1$, $\varepsilon_1 = 1$ – air, $\varepsilon_2 = 4$ – glass, curves 1,2,3 correspond to values $d = 10, 50, 100$, $\nu = 0.001\omega_p$, $0 \leq \theta \leq \pi/2$.

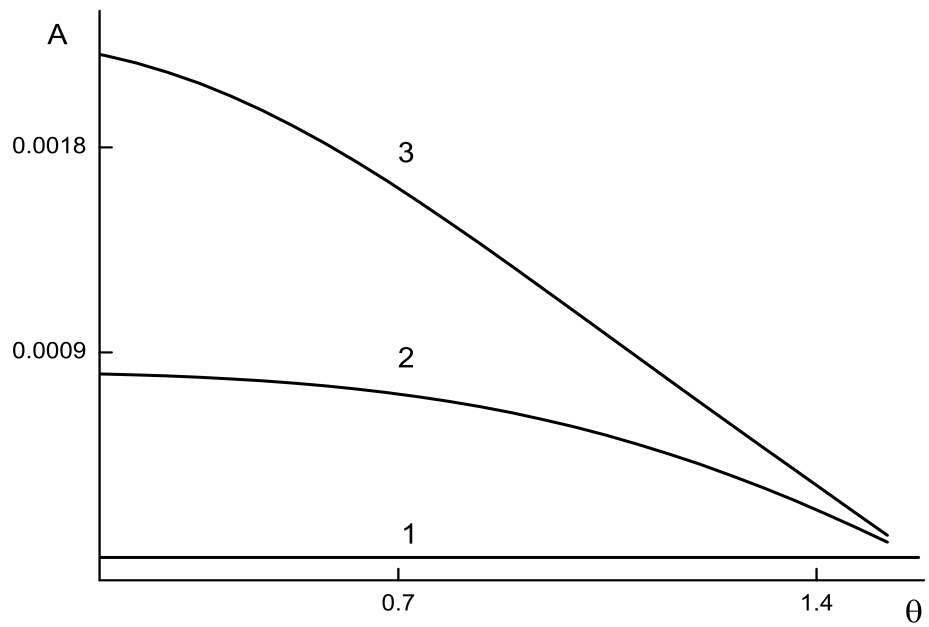


Figure 10. Absorptance, $\Omega = 1$, $\varepsilon_1 = 1$ – air, $\varepsilon_2 = 4$ – glass, curves 1,2,3 correspond to values $d = 10, 50, 100$, $\nu = 0.001\omega_p$, $0 \leq \theta \leq \pi/2$.

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